Homework Problems
Physics 417/517
Due February 11, 2015

1. An electron goes through 10 revolutions in a betatron with $n = 0.6$. How many vertical oscillations does it perform in the small displacement approximation? How many radial oscillations does it perform? Because the time-scale is so short for 10 revolutions, you may assume that γ is constant.

2. The so-called East Arcs in the CEBAF electron accelerator at Jefferson Lab consist of five separate beam-lines that operate at different electron beam energies. The rectangular dipoles in each beam-line are identical, are in regular configuration where the incident angle is one-half the total bend angle, and all together bend the beam through 180 degrees. More and longer magnets are needed to bend the higher beam energy electrons. When the total electron energy is 5500 MeV, the electron energy in each of the arcs is given in the table. With the information given, calculate the other entries in this table:

Arc	Electron Energy (MeV)	Number of Dipoles	Dipole Length (m)	Bend Angle (rad)	Magnetic Field (T)
1	605	16	1		
2	1693	32	1		
3	2781	32	2		
4	3868	32	3		
5	4956	32	3		

3. Assume the equation for an ellipse is
$$Ax^2 + 2Bxy + Cy^2 = D$$
where D has the units of area. Show, by performing the proper two dimensional integral, that the area of the ellipse is
$$\frac{\pi D}{\sqrt{AC - B^2}}.$$

4. As was done in class for the focusing direction, derive the transfer matrix for a region of length L with constant defocusing ($k < 0$)
$$M = \begin{pmatrix} \cosh\left(\sqrt{-k}L\right) & \sinh\left(\sqrt{-k}L\right)/\sqrt{-k} \\ \sqrt{-k}\sinh\left(\sqrt{-k}L\right) & \cosh\left(\sqrt{-k}L\right) \end{pmatrix}.$$
Evaluate the determinate of the transfer matrix.

5. A commonly applied focusing system in accelerators is the so-called FODO system. For a thin lens approximation to this system, the one period transfer matrix starting from the middle of the focusing lens is

$$M = \begin{pmatrix} 1 & 0 \\ -1/(2f) & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix} \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1/(2f) & 1 \end{pmatrix}.$$

where f is the lens focal length and L is the distance between lenses. Evaluate the total transfer matrix.

What is the result of a similar calculation to obtain the one period transfer map starting at the middle of the defocusing lens? (Hint: You don't have to perform the whole matrix multiplication again. Change a relevant parameter in the solution you've already obtained!).

Compare the matrix traces of the two results you have obtained.